

QUESTIONS OF UNITS AND MEASUREMENT

1) A STUDENT WRITES A FORMULA FOR THE TIME PERIOD 'T' OF A CONICAL PENDULUM AS

$$T = K \cdot \frac{H^a m^b}{g^c \cos^d \theta}$$

WHERE, H = RADIUS OF THE CIRCULAR PATH
 m = MASS OF THE BOB
 g = ACCELERATION DUE TO GRAVITY
 θ = ANGLE WITH VERTICAL
 K = DIMENSIONLESS CONSTANT

FIND THE VALUES OF a, b, c, d . USING DIMENSION ANALYSIS.

ANS)

DIMENSIONS OF EACH QUANTITY

$$H = [L] \quad T = [T]$$

$$m = [M] \quad \cos \theta = \text{DIMENSIONLESS}$$

$$g = [LT^{-2}]$$

$$\text{GIVEN, } T = K \cdot \frac{H^a m^b}{g^c \cos^d \theta} \Rightarrow T = \frac{[L]^a [M]^b}{([LT^{-2}])^c}$$

$$[M^0 T^1] = [L^{a-c} M^b T^{2c}]$$

COMPARING POWERS,

$$[M] \Rightarrow b = 0$$

$$[L] \Rightarrow a - c = 0 \Rightarrow a = c$$

$$[T] \Rightarrow 2c = 1 \\ c = \frac{1}{2}$$

FINAL ANSWERS

$$a = \frac{1}{2}$$

$$b = 0$$

$$c = \frac{1}{2}$$

d = any value ($\cos \theta$ has no dimension)

2) THE PRESSURE OF AN IDEAL GAS IS CALCULATED USING THE FORMULA:

$$P = \frac{nRT}{V}$$

Where:

$$n = 1.0 \pm 0.01 \text{ mol}$$

$$R = 8.31 \text{ J/mol-K (EXACT)}$$

$$T = 300 \pm 1 \text{ K}$$

$$V = 2.0 \pm 0.01 \text{ m}^3$$

FIND THE PERCENTAGE ERROR IN PRESSURE
ANS)

GIVEN:

$$P = \frac{nRT}{V} \Rightarrow \frac{\Delta P}{P} = \frac{\Delta n}{n} + \frac{\Delta T}{T} + \frac{\Delta V}{V}$$

NOTE:
R IS EXACT,
NO ERROR

INDIVIDUAL ERRORS:

$$\rightarrow \frac{\Delta n}{n} = \frac{0.01}{1.0} = 1\%$$

$$\rightarrow \frac{\Delta T}{T} = \frac{1}{300} \approx 0.333\%$$

$$\rightarrow \frac{\Delta V}{V} = \frac{0.01}{2.0} = 0.5\%$$

NOW TOTAL RELATIVE ERROR:

$$\begin{aligned} \frac{\Delta P}{P} &= \frac{\Delta n}{n} + \frac{\Delta T}{T} + \frac{\Delta V}{V} \\ &= 1\% + 0.333\% + 0.5\% \\ &= \underline{1.833\%} \end{aligned}$$

∴ PERCENTAGE ERROR IN PRESSURE = 1.833%

3) A PHYSICAL QUANTITY X IS GIVEN BY

$$X = \frac{A^2 B}{\sqrt{C}}$$

WHERE:

→ 'A' HAS DIMENSIONS = $[MLT^{-2}]$

→ 'B' HAS DIMENSIONS = $[L^2 T^{-1}]$

→ 'C' HAS DIMENSIONS = $[ML^{-1} T^{-2}]$

WHAT ARE THE DIMENSIONS OF X?

ANS) Given:

$$A = [MLT^{-2}] \Rightarrow A^2 = [M^2 L^2 T^{-4}]$$

$$B = [L^2 T^{-1}]$$

$$C = [ML^{-1} T^{-2}] \Rightarrow \sqrt{C} = [M^{1/2} L^{-1/2} T^{-1}]$$

$$X = \frac{A^2 B}{\sqrt{C}} = \frac{[M^2 L^2 T^{-4}] \cdot [L^2 T^{-1}]}{[M^{1/2} L^{-1/2} T^{-1}]}$$

$$= \frac{[M^2 L^4 T^{-5}]}{[M^{1/2} L^{-1/2} T^{-1}]}$$

$$= [M^{2-1/2} L^{4+1/2} T^{-5+1}]$$

$$= [M^{3/2} L^{9/2} T^{-4}]$$

DIMENSIONS OF X ARE

$$[M^{3/2} L^{9/2} T^{-4}]$$

Q 18 Potential energy given by 'U', Distance given by 'x' and time is given by 't'. Then find the dimensional formula of AxB. Also if AxB is given by $M^x L^y T^z$ then find the value of $|x| + |y| + |z|$. Equation: $U = \frac{At^2}{B+x^2}$

Sol: Potential energy = ML^2T^{-2}

Distance = L

Time = T

For finding B

We know only distance can be added with distance

$B+x^2$

$\therefore B = L^2$

For finding A

$$ML^2T^{-2} = \frac{AT^2}{L^2}$$

$$A = \frac{ML^4T^{-2}}{T^2} \Rightarrow A = ML^4T^{-4}$$

$$A \times B \Rightarrow ML^4T^{-4} \times L^2 \Rightarrow ML^6T^{-4}$$

$$x = 1, y = 6, z = -4$$

$$\therefore |x| + |y| + |z| = 6 + 4 + 1 = 11$$

Q 18 Force is given by 'F', velocity is given by 'v' and time is given by 't'. Find dimensional formula of $\frac{A}{B}$

$$\text{Equation: } F = \frac{Av^2}{B+t^2}$$

✱ Sol: For finding B

Only Time can be added with time

$$B + t^2$$

$$\therefore B = T^2$$

For finding A

$$F = MLT^{-2}$$

$$\Rightarrow MLT^{-2} = \frac{A \times L^2 T^{-2}}{T^2}$$

$$[v^2 = (LT^{-1})^2 = L^2 T^{-2}]$$

$$A = \frac{ML}{L^2 T^{-2}} \Rightarrow A = ML^{-1} T^2$$

$$\therefore \frac{A}{B} = \frac{ML^{-1} T^2}{T^2} = ML^{-1}$$

Q 18 Displacement is denoted by 'x' and time is denoted by 't'. Then find dimensional formula of A, B, C

$$\text{Equation: } x = A \cos(Bt^2 + cx^2)$$

Sol: Dimensional formula

$$x = L ; t = T$$

$$x = A \Rightarrow A = L$$

We know Angle (θ) is Dimensionless

$$\therefore Bt^2 = 1$$

$$B = T^{-2}$$

$$\therefore cx^2 = 1$$

$$C = \frac{1}{L^2} = L^{-2}$$

Q.1) The Expression given below shows variation of velocity (v) and time (t), $v = At^2 + \frac{Bt}{C+t}$. The dimension of ABC is?

* Answer:-

$$[LT^{-1}] = A[T^2] + \frac{B[T]}{C+[T]}$$

$$[LT^{-1}] = A[T^2]$$

$$\therefore A = LT^{-3}$$

$$\therefore C = [T] \left\{ \because \text{Since } C \text{ is added to } [T] \right\}$$

$$\frac{B[T]}{C+[T]} = [2T^{-1}]$$

$$\frac{B}{T} = 2T^{-1}$$

$$\therefore B = 2T^{-1}$$

$$\therefore ABC \Rightarrow [LT^{-3}][T][2T^{-1}]$$

$$\Rightarrow L^2 T^{-3}$$

Q.2) The pair of physical quantities not having same dimensions is:-

(1) Pressure and Young modulus (2) Surface Tension and Impulse

(3) Torque and Energy

(4) Angular Momentum and Planck's constant

* Answer: -

$$\text{Angular Momentum} = M L^2 T^{-1}$$

$$\text{Planck's constant} = M L^2 T^{-1}$$

$$\text{Torque} = M L^2 T^{-2}$$

$$\text{Energy} = M L^2 T^{-2}$$

$$\text{Surface Tension} = M T^{-2} \checkmark$$

$$\text{Impulse} = M L T^{-1} \checkmark$$

$$\text{Pressure} = M L^{-1} T^{-2}$$

$$\text{Young's Modulus} = M L^{-1} T^{-2}$$

$\therefore$ Option (2) is the right answer

Q. 3) If B is magnetic field and μ_0 is permeability of free space, then the dimensions of (B/μ_0) is?

* Answer: -

ATQ, we have B which is magnetic field and μ_0

which is permeability of free space.

We know that there is equation which connects these:-

$$B = \frac{\mu_0 i}{2R} \quad \left\{ \begin{array}{l} \text{B} = \frac{\mu_0 i}{2R} \\ \text{B} = \frac{\mu_0 n i}{2R} \end{array} \right\} \text{ i - current}$$

$$\frac{B}{\mu_0} = \frac{i}{2R} \quad \frac{B}{\mu_0} = n i = \underline{\underline{[L^{-1} A]}} \quad \left\{ \begin{array}{l} B = M T^{-2} A^{-1} \\ \mu_0 = M L T^{-2} I^{-2} \end{array} \right\}$$

Units and Measurements

Dhaa-a
2025-26

Q1. If the energy E , velocity V , and time T are chosen as the fundamental units, what will be the dimensional formula for surface tension?

Ans. Let the dimensional formula for surface tension $[S]$ in terms of energy, velocity and time be:

$$S = E^x V^y T^z$$

Standard dimensional formulas for these quantities in terms of mass $[M]$, length $[L]$, and time $[T]$:

→ Surface tension $[S]$ = Force per unit length = $\frac{\text{Force}}{\text{Length}}$

$$= \frac{MLT^{-2}}{L}$$

$$= MT^{-2}$$

→ Energy $[E]$ = Work done
= Force $\times$ Distance
= $MLT^{-2} \times L$
= ML^2T^{-2}

→ Velocity $[V]$ = $\frac{\text{Length}}{\text{Time}}$
= LT^{-1}

→ Time $[T]$ = T

Substituting these values in the equation for S :

$$MT^{-2} = (ML^2T^{-2})^x (LT^{-1})^y (T)^z$$

$$MT^{-2} = M^x L^{2x} T^{-2x} L^y T^{-y} T^z$$

$$MT^{-2} = M^x L^{2x+y} T^{-2x-y+z}$$

Now, comparing the powers of M, L, T on both sides:

For M : $x = 1$

For L : $2x + y = 0$

$$2(1) + y = 0$$

$$\underline{y = -2}$$

$$\begin{aligned}\text{For } T: -2x - y + z &= -2 \\ -2(1) - (-2) + z &= -2 \\ -2 + 2 + z &= -2 \\ \underline{\underline{z = -2}}\end{aligned}$$

$\therefore$ The dimensional formula for surface tension is $\underline{\underline{L^1 V^{-2} T^{-2}}}$.

Q2. The dimensions of stopping potential V_0 in photoelectric effect in units of Planck's constant ' h ', speed of light ' c ' and gravitational constant ' G ' and ampere A is?

Ans: Stopping potential (V_0) $\propto h^x A^y G^z C^w$

Dimensional formulas:

$$\text{Planck's constant} = h = [ML^2 T^{-1}]$$

$$\text{Current} = I = [A]$$

$$\text{Gravitational constant} = G = [M^{-1} L^3 T^{-2}]$$

$$\text{Speed of light} = c = [L T^{-1}]$$

$$\text{Potential} = V_0 = [ML^2 T^{-3} A^{-1}]$$

$$\begin{aligned}\therefore ML^2 T^{-3} A^{-1} &= [ML^2 T^{-1}]^x [A]^y [M^{-1} L^3 T^{-2}]^z [L T^{-1}]^w \\ &= M^x L^{2x} T^{-1x} A^y M^{-1z} L^{3z} T^{-2z} L^w T^{-1w}\end{aligned}$$

$$ML^2 T^{-3} A^{-1} = M^{x-z} L^{2x+3z+w} T^{-x-2z-w} A^y$$

Comparing dimensions of M, L, T & A ,

$$y = -1$$

$$M \Rightarrow 1 = x - z$$

$$\Rightarrow \underline{\underline{x = 1 + z}}$$

$$L \Rightarrow 2x + 3z + w = 2$$

$$2 + 2z + 3z + w = 2$$

$$2 + 5z + w = 2$$

$$\underline{\underline{w = -5z}}$$

$$T \Rightarrow -x - 2z - r = -3$$

$$-1 - z - 2z + 5z = -3$$

$$-2 = 2z$$

$$z = -1$$

$$\therefore x = 5$$

$$x = 0$$

$$\therefore \underline{V_0 \propto h^0 A^{-1} G^{-1} C^5}$$

Q3. The speed of a wave produced in water is given by $V = \lambda^a g^b \rho^c$. Where λ , g and ρ are wavelength of wave, acceleration due to gravity and density of water respectively. Find the values of a , b , c .

Ans. Dimensional Formulas:

$$V = [L T^{-1}]$$

$$\lambda = L$$

$$g = L T^{-2}$$

$$\rho = M L^{-3}$$

$$V = \lambda^a g^b \rho^c$$

$$\Rightarrow L T^{-1} = [L]^a [L T^{-2}]^b [M L^{-3}]^c$$

$$L T^{-1} = [M^c L^{a+b-3c} T^{-2b}]$$

On Comparing respective powers,

$$\underline{c=0}$$

$$T: -2b = -1$$

$$b = \underline{\underline{\frac{1}{2}}}$$

$$L: a + \frac{1}{2} - 3(0) = 1$$

$$a = \underline{\underline{\frac{1}{2}}}$$

$\therefore$ Value of a , b and c are $\underline{\underline{\frac{1}{2}}}$, $\underline{\underline{\frac{1}{2}}}$ and c respectively.

UNITS AND MEASUREMENTS

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Q.1

Q1. If the unit of Force is 1 kN, the unit of length is 1 km and the unit of time is 1 minute, what will be the unit of mass?

→ By Newton's 2nd law, Force = Mass × Acceleration

$$\Rightarrow F = ma$$

$$\Rightarrow m = \frac{\text{Force}}{\text{Acceleration}}$$

$$\text{Acceleration} = \frac{\text{length}}{(\text{Time})^2}$$

$$\Rightarrow m = \frac{\text{Force} \times (\text{Time})^2}{\text{length}}$$

Now, we know.

$$\begin{aligned}\text{Force } (F_{\text{new}}) &= 1 \text{ kN} = \underline{1000 \text{ N}} \\ \text{length } (L_{\text{new}}) &= 1 \text{ km} = \underline{1000 \text{ m}} \\ \text{Time } (T_{\text{new}}) &= 1 \text{ minute} = \underline{60 \text{ s}}\end{aligned}$$

Substituting these into the derived formula for mass:

$$M_{\text{new}} = \frac{F_{\text{new}} \times (T_{\text{new}})^2}{L_{\text{new}}}$$

$$\Rightarrow M_{\text{new}} = \frac{1000 \text{ N} \times (60)^2}{1000 \text{ m}}$$

$$\Rightarrow \underline{M_{\text{new}} = 3600 \text{ kg}}$$

Q2 A student measures the diameter of a wire using a screw gauge. The pitch of the screw gauge is 1 mm & there're 100 divisions on the circular scale. Before starting the measurement, it's found that when the jaws of the screw gauge are brought in contact, the 5th division of the circular scale is below the reference line and the main scale reading is 0 mm. When the wire is placed between the jaws, main scale reading is 2 mm and the 30th division of the circular scale coincides with the reference line. The diameter of the wire is —

Least Count Of Screw Gauge (LC) = $\frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$= \frac{1 \text{ mm}}{100} = \underline{0.01 \text{ mm}}$$

Since the 5th division of the circular scale is below the reference line, there's a positive zero error.

$$\text{Zero Error (ZE)} = (\text{Circular Scale Division} \times \text{LC})$$

$$= 5 \times 0.01 = \underline{+0.05 \text{ mm}}$$

Now, readings when measuring the wire:

$$\text{Main Scale Reading} = \text{MSR} = \underline{2 \text{ mm}}$$

$$\text{Circular Scale Reading} = \text{CSR} = \underline{30}$$

$$\text{Observed Reading} = \text{MSR} + (\text{CSR} \times \text{LC}) = 2 \text{ mm} + (30 \times 0.01 \text{ mm})$$

$$= \underline{2.30 \text{ mm}}$$

$$\text{Corrected Reading} = \text{Observed reading} - \text{Zero Error}$$

$$= 2.30 \text{ mm} - (+0.05 \text{ mm}) = \underline{2.25 \text{ mm}}$$

$$\therefore \text{Diameter Of The Wire} = \underline{2.25 \text{ mm}}$$

Q3.

In a system of units, the units of mass is A kg, length is B meters, and time is C seconds. The joule's value in this system will be.

→ Joule in the S.I. system is defined as $\text{kg m}^2 \text{s}^{-2}$

let n_1 be the numerical value of 1 Joule in S.I. system. $\Rightarrow n_1 = 1$

let U_1 be S.I. Unit $\Rightarrow U_1 = \text{kg m}^2 \text{s}^{-2}$

let n_2 be numerical value in new system of units.

let U_2 be the new unit.

The new unit of mass = $M_2 = A \text{ kg}$ [$M_1 = 1 \text{ kg}$]

The new unit of length = $L_2 = B \text{ meter}$ [$L_1 = 1 \text{ m}$]

The new unit of Time = $T_2 = C \text{ sec}$ [$T_1 = 1 \text{ sec}$]

We know, $n_1 U_1 = n_2 U_2$

Dimensions of Energy = $\underline{ML^2T^{-2}}$

$$\Rightarrow n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c, \text{ where } a=1, b=2, c=-2$$

$$\Rightarrow n_2 = 1 \left(\frac{1 \text{ kg}}{A \text{ kg}} \right)^1 \left(\frac{1 \text{ m}}{B \text{ m}} \right)^2 \left(\frac{1 \text{ sec}}{C \text{ sec}} \right)^{-2}$$

$$\Rightarrow n_2 = \frac{1}{A} \times \frac{1}{B^2} \times \frac{1}{C^2}$$

$$\Rightarrow n_2 = \frac{1}{A} \times \frac{1}{B^2} \times C^2$$

$$\Rightarrow n_2 = \frac{C^2}{AB^2}$$

$$\Rightarrow n_2 = \frac{1}{AB^2C^{-2}}$$

UNITS AND DIMENSIONS

Q1) The force is given in terms of time t and displacement x by equation: $F = A \cos Bx + C \sin Dt$
The dimensional formula of $\frac{AD}{B}$ is:?

Sol:

$$F = MLT^{-2}$$

$$F = A \cos Bx + C \sin Dt$$

$$\text{So, } F = A \cos Bx = C \sin Dt = MLT^{-2} \quad \left\{ \text{We know, } F = MLT^{-2} \right\}$$

$$A \cos Bx = MLT^{-2}$$

Since $\cos Bx$ is constant

$$\boxed{A = MLT^{-2}}$$

$$\text{also, } \cos Bx = L^0 \quad \left\{ \text{trigonometric function is constant} \right\}$$

$$B L^0 = L^0$$

$$\boxed{B = L^{-1}}$$

Also,

$$C \sin Dt = MLT^{-2}$$

Since $\sin Dt$ is constant

$$C = MLT^{-2}$$

$$\sin Dt = L^0 \quad \left\{ \text{sin function is constant} \right\}$$

$$D T^0 = L^0$$

$$\boxed{D = T^{-1}}$$

$$\frac{AD}{B} = \frac{MLT^{-2} T^{-1}}{L^{-1}} = \boxed{ML^2 T^3}$$

Q2) An Vander waals equation $\left[P + \frac{a}{V^2}\right][V-b] = RT$, P is pressure V is Volume, R is universal gas constant and T is temperature. The ratio of $\frac{a}{b}$ is dimensionally equal to what?

Sol:

$$RT = \left[P + \frac{a}{V^2}\right][V-b]$$

$$P + \frac{a}{V^2} = mL^{-1}T^{-2} + \frac{a}{L^3} = mL^{-1}T^{-2} \left\{ P = mL^{-1}T^{-2} \text{ \& } V = L^3 \right\}$$

$$\frac{a}{L^3} = mL^{-1}T^{-2}$$

$$\boxed{a = mL^5T^{-2}}$$

$$[V-b]$$

$$L^3 - b = L^3$$

$$\boxed{b = L^3}$$

$$\frac{a}{b} = \frac{mL^5T^{-2}}{L^3} = mL^2T^{-2}$$

$$\frac{a}{b} = mL^2T^{-2} = mL^{-1}T^{-2} \times L^3 = P \times V = \boxed{PV}$$

Q3) To find distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimension analysis and assumes that distance depends on mass density ρ of fog, intensity (power/area) S of light from signal and its ~~over~~ frequency f . The engineer finds that d is proportional to S^n . The value of n is.

Sol.

$d \propto \rho^a \times s^b \times f^c$ {considered, a, b, c are power of density, intensity and frequency respectively}

$$d = k \rho^a \times s^b \times f^c$$

$$L = (ML^{-3})^a \times \left(\frac{MLT^{-3}}{L^2} \right)^b \times [T^{-1}]^c \left\{ \begin{array}{l} \rho = ML^{-3}, \mu_{\text{area}} = ML^2T^{-3}, A = L^2 \\ f = T^{-1} \end{array} \right\}$$

$$M^0 L^1 T^0 = M^{a+b} \cdot L^{-3a} \cdot T^{-3b-c}$$

$$-3a = 0$$

$$\boxed{a = 0 - \frac{1}{3}}$$

$$a+b=0$$

$$-\frac{1}{3} + b = 0$$

$$\boxed{b = \frac{1}{3}}$$

$$s^b = s^{\frac{1}{n}}$$

$$b = \frac{1}{n}$$

$$\frac{1}{3} = \frac{1}{n}$$

$$\boxed{\text{So } n = 3}$$

Units and Dimension

Q1) A physical quantity Q is related to your Observables a, b, c and d as follows:

$$Q = \frac{ab^4}{cd}$$

$a = (60 \pm 3) \text{ Pa}$; $b = (20 \pm 0.1) \text{ m}$; $c = (40 \pm 0.2) \text{ Nsm}^{-2}$;
 $d = (50 \pm 0.1) \text{ m}$, then the percentage error in Q is $\frac{x}{1000}$,
where $x = \underline{7700}$.

Ans:-

Given,

$$Q = \frac{ab^4}{cd} \quad \left| \begin{array}{l} a = (60 \pm 3) \text{ Pa} \Rightarrow a = 60 \text{ Pa}; \Delta a = 3 \text{ Pa} \\ b = (20 \pm 0.1) \text{ m} \Rightarrow b = 20 \text{ m}; \Delta b = 0.1 \text{ m} \\ c = (40 \pm 0.2) \text{ Nsm}^{-2} \Rightarrow c = 40 \text{ Nsm}^{-2}; \Delta c = 0.2 \text{ Nsm}^{-2} \end{array} \right.$$

$$d = (50 \pm 0.1) \text{ m} \Rightarrow d = 50 \text{ m}; \Delta d = 0.1 \text{ m}$$

$$\text{As, } Q = \frac{ab^4}{cd}$$

$$\Rightarrow \log Q = \log a + 4 \log b - \log c - \log d$$

$$\Rightarrow \frac{dQ}{Q} = \frac{da}{a} + 4 \frac{db}{b} - \frac{dc}{c} - \frac{dd}{d}$$

$$\Rightarrow \frac{\Delta Q}{Q} = \frac{\Delta a}{a} + 4 \frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\Rightarrow \frac{\Delta Q}{Q} = \frac{3}{60} + 4 \times \frac{0.1}{20} + \frac{0.2}{40} + \frac{0.1}{50}$$

$$\Rightarrow \frac{\Delta Q}{Q} = \frac{77}{1000} \quad \left| \therefore Q = \frac{\Delta Q}{Q} \times 100\% \right.$$

$$\Rightarrow \frac{7700}{1000} = \frac{x}{1000} \Rightarrow \boxed{x = 7700}$$

② In an electromagnetic system, the quantity representing the ratio of electric flux and magnetic flux has dimensions of $M^P L^Q T^R A^S$, where value of 'Q' and 'R' are:

- Ⓐ (3, -5) Ⓑ (-2, 1) Ⓒ (-2, 2) Ⓓ (1, -1)

Ans:-

Electric flux is defined as:-

$$\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{A}$$

$$\therefore [E] = \frac{\text{Force}}{\text{Charge}} \Rightarrow [E] = \frac{MLT^{-2}}{AT} = MLT^{-3}A^{-1}$$

$$\therefore [\Phi_E] = [E][A] = MLT^{-3}A^{-1} \cdot L^2 = ML^3T^{-3}A^{-1}$$

$$\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{A}$$

$$[B] = \frac{F}{qV} = \frac{MLT^{-2}}{(AT)(T^{-1}L)} = MT^{-2}A^{-1}$$

$$\therefore \Phi_B = [B][A] = ML^2T^{-2}A^{-1}$$

$$\therefore \frac{\Phi_E}{\Phi_B} = \frac{ML^3T^{-3}A^{-1}}{ML^2T^{-2}A^{-1}} = LT^{-1}$$

$$\therefore Q=1 \quad R=-1 \quad [\text{D}]$$

③ If μ_0 and ϵ_0 are the permeability and permittivity of free space respectively, the dimension of $\left(\frac{1}{\mu_0 \epsilon_0}\right)$ is:

- (A) $\frac{T^2}{L}$ (B) $\frac{L^2}{T^2}$ (C) $\frac{T^2}{L^2}$ (D) $\frac{L}{T^2}$

Ans:-

The expression $\frac{1}{\mu_0 \epsilon_0}$ is related to the speed of light (c), given by the expression,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$\therefore$ Speed of light (c) $\rightarrow LT^{-1}$

$$\therefore \frac{1}{\mu_0 \epsilon_0} = (LT^{-1})^2 \Rightarrow [L^2 T^{-2}] = \frac{1}{\mu_0 \epsilon_0}$$

Hence, the dimension is $\frac{L^2}{T^2}$ [(B)]

UNITS AND DIMENSIONS

(1) If velocity v , acceleration a , and time t are taken as fundamental quantities, what is the dimension of force F in terms of v, a, t ?

(a) $va^{-1}t$ (b) vat (c) vat^2 (d) vat^{-1}

Solution :-

$$F = ma$$

mass can be expressed of v, a, t

$$[v] = LT^{-1}, [a] = LT^{-2}, [t] = T$$

Rearranging

$$F = v \cdot a \cdot t$$

Ans :- (B) vat //

(2) The dimensional formula of Planck's

constant is :

(A) $M^1 L^2 T^{-1}$ (B) $M^1 L^2 T^{-2}$ (C) $M^1 L^1 T^{-2}$

Solution $E = hv$ $E = ML^2 T^{-2}$

$$v = T^{-1}$$

$$[h] = \frac{E}{v} = \frac{ML^2 T^{-2}}{T^{-1}} = ML^2 T^{-1}$$

option (A) $M^1 L^2 T^{-1}$

(3) If g is acceleration due to gravity and R is radius of Earth, then the time period of a satellite orbiting close to Earth's surface is proportional to
(A) R/g (B) $R^{3/2}/\sqrt{g}$ (C) $\sqrt{g/R}$ (D) $\sqrt{R/g}$

Solution:-

Orbital time period near Earth

$$T = 2\pi \sqrt{\frac{R}{g}}$$

Therefore, $T \propto \sqrt{\frac{R}{g}}$

$$(D) \sqrt{R/g}$$

Q.1 Calculate the value of force 100 dynes on a system based on metre, kilogram and minute as fundamental units.

Ans Dyne is the unit of force in C.G.S system
D.F for force = $[MLT^{-2}]$

Here, $a=1, b=1, c=2$

In C.G.S system, $M_1=1g, L_1=1cm, T_1=1s$ and $n_1=100$

In New system, $M_2=1kg, L_2=1m, T_2=1min$ and $n_2=?$

We know that, $n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$

$$= 100 \times \left(\frac{1g}{1kg} \right)^1 \left(\frac{1cm}{1m} \right)^1 \left(\frac{1s}{1min} \right)^{-2}$$

$$= 100 \times \left(\frac{1g}{1000g} \right)^1 \left(\frac{1cm}{1000cm} \right)^1 \left(\frac{1s}{60s} \right)^{-2}$$

$$= 100 \times \frac{1}{1000} \times \frac{1}{1000} \times 3600 = 3.6$$

Q.2 Convert Joule into erg

Ans Joule is unit of work in S.I and erg in C.G.S system

S.I unit, $M_1=1kg, L_1=1m, T_1=1s$ and $n_1=1$

C.G.S system, $M_2=1g, L_2=1cm, T_2=1s$ and $n_2=?$

$$n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$$

The D.F of work is $[ML^2T^{-2}]$

$\therefore a=1, b=2$ and $c=-2$

The number of erg in 1J is given by

$$n_2 = 1 \times \left(\frac{1 \text{ kg}}{1 \text{ g}} \right)^1 \left(\frac{1 \text{ cm}}{1 \text{ m}} \right)^2 \left(\frac{1 \text{ s}}{1 \text{ s}} \right)^{-2}$$

$$= \left(\frac{1000 \text{ g}}{1 \text{ g}} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = 10^7$$

$$1 \text{ J} = 10^7 \text{ erg}$$

Q.3 What is the unit of pressure if the unit of mass is kg, the unit of length is metre and unit of time is minute?

ans D.F for pressure is $[ML^{-1}T^{-2}]$

$$x=1, y=-1 \text{ and } z=-2$$

$$n_2 = n_1 \left[\left(\frac{m_1}{m_2} \right)^x \left(\frac{L_1}{L_2} \right)^y \left(\frac{T_1}{T_2} \right)^z \right]$$

$$1 = n_1 \left(\frac{1 \text{ g}}{1 \text{ kg}} \right)^1 \left(\frac{1 \text{ cm}}{1 \text{ m}} \right)^{-1} \left(\frac{1 \text{ s}}{1 \text{ min}} \right)^{-2}$$

$$1 = n_1 \left(\frac{1}{1000} \right)^1 \left(\frac{1}{100} \right)^{-1} \left(\frac{1}{60} \right)^{-2}$$

$$n_1 = (1000)^1 (100)^{-1} (60)^{-2}$$

$$= \frac{1000}{100 \times 60 \times 60} = \frac{1}{360}$$

$$\therefore \text{Unit of pressure} = \frac{1}{360} \text{ dyne/cm}^2$$

UNITS & Measurements

Q1. Find the dimensions of the quantity

$$\frac{\text{Force} \times \text{velocity}}{\text{Power}}$$

Sol:

$$\text{Force} = [MLT^{-2}]$$

$$\text{Velocity} = [LT^{-1}]$$

$$\text{Power} = [ML^2T^{-3}]$$

$$\frac{[MLT^{-2}] \cdot [LT^{-1}]}{[ML^2T^{-3}]} = \frac{ML^2T^{-3}}{ML^2T^{-3}} = [1]$$

The quantity is dimensionless.

Q2. If $A = 2.0 \pm 0.1$ and $B = 3.0 \pm 0.2$, find the Percentage error in $C = A \times B$.

~~Q2~~
Solution:

$$\frac{\Delta C}{C} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$\frac{\Delta A}{A} = \frac{0.1}{2.0} = 0.05 = 5\%$$

$$\frac{\Delta B}{B} = \frac{0.2}{3.0} \approx 0.0667 = 6.67\%$$

$$5\% + 6.67\% = \boxed{11.67\%}$$

Q The time period T of a simple pendulum depends on length " l ", acceleration due to gravity " g " and a dimensionless constant " k ". Using dimensional analysis, find the form of T .

Sol:

$$T = k l^a g^b$$

$$[T] = [L]^a \cdot [LT^{-2}]^b$$

$$[T] = [L^{a+b} T^{-2b}]$$

$$a+b=0,$$

$$-2b=1 \Rightarrow b = -\frac{1}{2}, \quad a = \frac{1}{2}.$$

$$T = k \sqrt{\frac{L}{g}}.$$

$$T \propto \sqrt{\frac{L}{g}}.$$

- Q. Find dimensional formula of the universal gravitational constant G in the formula:

$$F = G \frac{m_1 m_2}{r^2}$$

$$\text{Sol: } F = G \frac{m_1 m_2}{r^2} \Rightarrow G = \frac{F r^2}{m_1 m_2}$$

$$F \rightarrow \text{Force} \rightarrow [M L T^{-2}]$$

$$r \rightarrow \text{distance} \rightarrow [L]$$

$$m \rightarrow \text{mass} \rightarrow [M]$$

$$G = \frac{[M L T^{-2}] [L]^2}{[M]^2} = \frac{[M L^3 T^{-2}]}{[M]^2} = [M^{-1} L^3 T^{-2}]$$

$\therefore$ dimensional formula is:

$$G = [M^{-1} L^3 T^{-2}]$$

- Q. The energy E of a particle is said to depend on its velocity v , mass m , & plank's constant h such that $E = k m^a v^b h^c$ using dimensional analysis find a, b, c .

$$\text{Sol: } [E] = [M L^2 T^{-2}]$$

$$[m] = [M]$$

$$[v] = [L T^{-1}]$$

$$[h] = [M L^2 T^{-1}]$$

$$\begin{aligned} \therefore [M L^2 T^{-2}] &= [M]^a [L T^{-1}]^b [M L^2 T^{-1}]^c \\ &= M^{a+c} \cdot L^{b+2c} \cdot T^{-b-c} \end{aligned}$$

Evaluating:

$$M = a+c=1, \quad L = b+2c=2$$

$$\therefore b+c=2 \rightarrow b=2-c$$

$$(2-c) + 2c = 2$$

$$= 2+c=2$$

$$c=0$$

$$b=2$$

$$a=1, b=2, c=0.$$

- Q. The eq of motion is given by: $x = ut + \frac{1}{2}at^2 + bt^3$ where x is displacement, u is initial velocity, a is acceleration, b is a constant. Check whether the eq is dimensionally consistent & find the dimensional formula of b .

Sol: LHS = x (displacement)

RHS = ut :

$$u = [LT^{-1}], \quad t = [T] \Rightarrow ut = [L]$$

$\frac{1}{2}at^2$:

$$a = [LT^{-2}], \quad t^2 = [T^2] \Rightarrow at^2 = [L]$$

bt^3 :

$$[b][T^3] = [L] \Rightarrow [b] = \frac{[L]}{[T^3]} = [LT^{-3}]$$

$\therefore$ Eq is dimensionally consistent & dimensionally formula of b is:

$$b = [LT^{-3}]$$

Q. Dimension of universal gravitation constant in terms of Planck's constant (h), distance (L), mass (M) and time (t).

(A) $[h T L M^{-2}]$

(B) $[h T^{-1} L M^{-2}]$

(C) $[h T L^2 M^{-2}]$

(D) $[h^{-1} T^{-1} L M^{-2}]$

Ans:

$$[G] = [M^{-1} L^3 T^{-2}]$$

$$E = h\nu$$

$$h = [M L^2 T^{-1}]$$

$$G = [h]^a [L]^b [M]^c [T]^d$$

Ans $\rightarrow [h T^{-1} L M^{-2}]$

Q.) Keeping significant figures in view, the sum of physical quantities 52.01 m, 153.2 m and 0.123 m is ____

(A) 205.3 m

(B) 205.33 m

(C) 205.33 m

(D) 205 m

Ans

(A) 205.3 m

Q.) The percentage error in calculated Volume of a Sphere, if there's 2% error in its diameter measurement is _____

(A) 1

(B) 2

(C) 6

(D) 8

Ans

$$V = \frac{\pi}{6} D^3$$

$$\ln V = \ln \frac{\pi}{6} + 3 \ln D$$

$$\frac{\Delta V}{V} = 3 \frac{\Delta D}{D}$$

$$\frac{\Delta V}{V} \times 100 = 3 \left(\frac{\Delta D}{D} \times 100 \right)$$

$$\frac{\Delta D}{D} \times 100 = 2$$

$$\boxed{3 \times 2 = 6\%}$$

Q. An equation of velocity is given as:

$$v = at + bt^2$$

'a' and 'b' are constant, if the equation is dimensionally correct. Find dimensions of a & b

$$\Rightarrow v = at + bt^2$$

$$[v] = [LT^{-1}]$$

$$at \Rightarrow [a][t] = [LT^{-1}]$$

$$\underline{a = [LT^{-2}]} \longrightarrow (a)$$

$$bt^2 \Rightarrow [b][t^2] = [LT^{-1}]$$

$$\underline{b = [LT^{-3}]} \longrightarrow (b)$$

Q. A planet's orbit is measured as

$$r(\text{radius}) = (7.5 \pm 0.1) \times 10^8 \text{ m}$$

$$T(\text{time period}) = (8.16 \pm 0.1) \times 10^7 \text{ s}$$

Mass of sun formula is given as:

$$M = \frac{4\pi^2 r^3}{GT^2}$$

(Assume 'G' has no error)

Calculate % error in M.

$$\Rightarrow r^3 \longrightarrow \text{error} = 3 \times \frac{\Delta r}{r}$$

$$T^2 \longrightarrow \text{error} = 2 \times \frac{\Delta T}{T}$$

$$\frac{\Delta z}{z} = \frac{0.1}{7.5} = 0.0133$$

$$\frac{\Delta T}{T} = \frac{0.01}{3.15} = 0.00317$$

$$\frac{\Delta M}{M} = 3 \times 0.0133 + 2 \times 0.00317$$

$$= 0.0399 + 0.00634$$

$$= 0.04624$$

percentage error in M is $\rightarrow \boxed{4.62\%}$

Q Energy 'E', speed 'v' and time 't' are taken as fundamental quantities instead of $[M^0 L^0 T^0]$

Write dimensional formula of Mass in terms of $[E^a v^b t^c]$

$$\Rightarrow M = E^a v^b t^c$$

$$M = L^a M^1 T^0$$

$$E = M L^2 T^{-2}$$

$$v = L T^{-1}$$

$$t = T^1$$

$$E^a v^b t^c = (M^a L^{2a} T^{-2a}) (L^b T^{-b}) (T^c)$$

$$= M^a L^{2a} T^{-2a-b+c}$$

$$M \Rightarrow a = 1$$

$$L \Rightarrow 2a + b = 0$$

$$b = -2$$

$$T \Rightarrow -2a - b + c = 0$$

$$c = 0$$

$$M = E^1 v^{-2} t^0$$

$$\boxed{M = \frac{E}{v^2}}$$